End of Year Examination Paper 1

INSTRUCTION TO CANDIDATES:

Marks Obtained
80

1. Answer **all** questions.
2. Write your answers and working in the spaces provided.
3. Omission of essential working will result in loss of marks.
4. Calculators may be used in this paper.
5. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer correct to three significant figures. Give answers in degrees correct to one decimal place.

Duration: 2 hours

1 Simplify the the expression $\sqrt[3]{a^9b^3} \times (4a^8)^0$, leaving your answers in positive indices.

Ans: _____ [2]

2 Write down the gradient and y-intercept of the straight line $y = \frac{8x - 3}{5}$.

Ans: Gradient = _____ [1]

y-intercept = _____ [1]

3 (a) Express $\frac{0.00149}{100\,000}$ in standard form correct to 2 significant figures.

(b) DNA are complex molecules that store biological information. Each DNA stands about 2 nanometres in width. Express, in standard form,

(i) the width of a DNA strand in micrometres, μm,

(ii) the number of DNA strands, arranged side by side, to form a length of 5 cm.

(1 nanometre = 1×10^{-9} m; 1 micrometre = 1×10^{-6} m)

Ans: (a) _____ [2]

(b)(i) _____ μm [1]

(ii) _____ [2]

4 Solve the following equations.

(a) $x(x + 1) = 2$

(b) $\frac{3}{a-1} + \frac{4a}{1-3a} = 0$

Ans: (a) $x =$ _____ or _____ [3]

(b) $a =$ _____ or _____ [3]

5 Solve the following equations.

(a) $\frac{1}{4^x} = 2(16^{-x})$

(b) $\sqrt{3^y} \times 9^{y-1} \times \frac{1}{27^y} = 81$

Ans: (a) $x =$ _____ [2]

(b) $y =$ _____ [3]

6 (a) Solve the inequality $3 - (5 - 2x) \geqslant 25$.

(b) Write down the smallest prime number x which satisfies the inequality $3 - (5 - 2x) \geqslant 25$.

Ans: (a) _____ [2]

(b) $x =$ _____ [1]

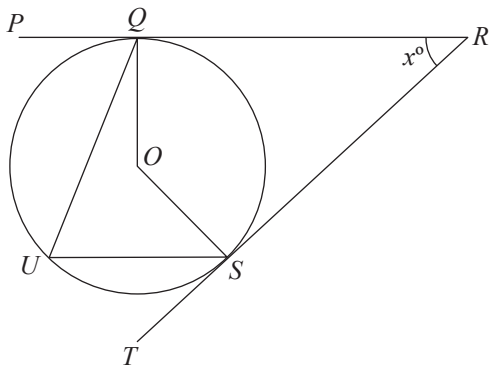
7 Q, S and U are points on the circle with centre O. PQR and TSR are tangents to the circle and $\angle PRT = x^\circ$. $\angle OSU$ is twice the size of $\angle OQU$.

Find, in terms of x,

(a) obtuse $\angle QOS$,

(b) $\angle QUS$,

(c) $\angle OQU$.



Ans: (a) $\angle QOS =$ _____ [1]

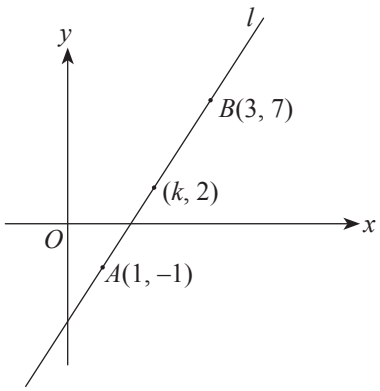
(b) $\angle QUS =$ _____ [1]

(c) $\angle OQU =$ _____ [2]

8 Line l passes through the points $A(1, -1)$ and $B(3, 7)$.

(a) Find the gradient of the line l.

(b) Line l also passes through a point $(k, 2)$. Find the value of k.



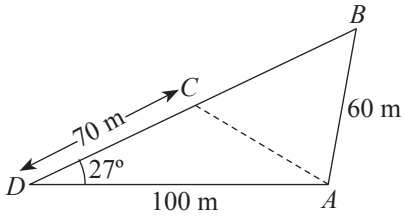
Ans: (a) _____ [1]

(b) k = _____ [2]

9 The diagram shows a triangular field ABD. AC is a walking path within the field. It is given that $AB = 60$ m, $AD = 100$ m, $CD = 70$ m and $\angle ADC = 27°$.

(a) Calculate the length of the path AC.

(b) Find the angle ABC.



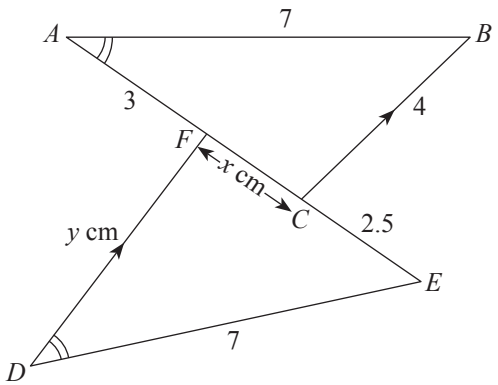
Ans: (a) AC = _____ m [2]

(b) $\angle ABC$ = _____ ° [2]

10 Solve the simultaneous inequality $-\frac{1}{4}x - 3 \geqslant 2 - 3x \geqslant x - 10$.
Show your answer on a number line.

Ans: on answer space [3]

11 The two triangles shown in the diagram are congruent. Given that $AB = ED = 7$ cm, $BC = 4$ cm, $AF = 3$ cm, $EC = 2.5$ cm and $\angle BAC = \angle FDE$ and $DF // CB$,



(a) state the pair of congruent triangles,
(b) find the value of x and of y.

Ans: (a) _____ ≡ _____ [1]

(b) $x =$ _____

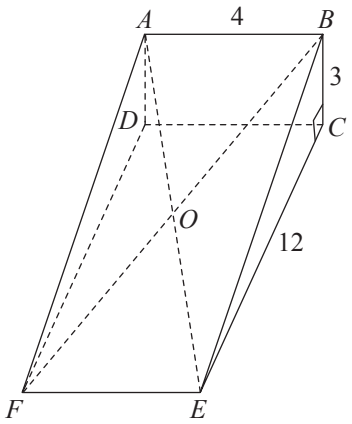
$y =$ _____ [4]

12 The diagram shows a wooden door wedge in the shape of a triangular prism such that $\angle ADF = \angle BCE = 90^\circ$ and $ABCD$ is a rectangle. It is given that $AB = 4$ cm, $BC = 3$ cm and $CE = 12$ cm. Calculate

(a) the length of CF,

(b) $\angle BFC$,

(c) $\angle AOB$, where O is the intersection of the diagonals AE and BF.



Ans: (a) $CF =$_____ cm [2]

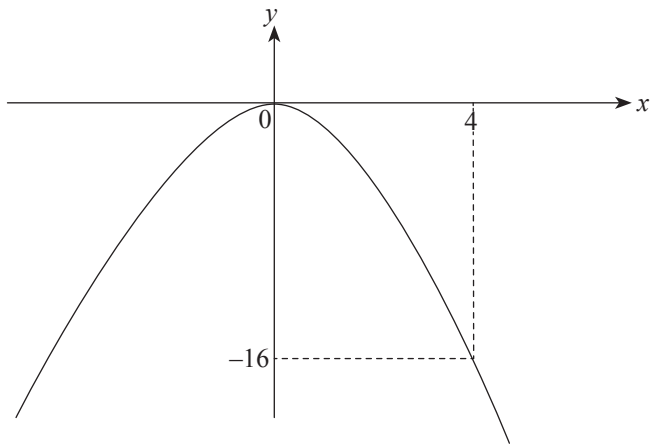
(b) $\angle BFC =$ _____ ° [1]

(c) $\angle AOB =$ _____ ° [3]

13 Mr Tan deposited \$86 000 into a savings account that pays simple interest at a rate of 1.15% per annum. Find the amount that Mr Tan has in the savings account after 12 years.

Ans: \$ _____ [2]

14 The diagram shows a quadratic curve of the equation $y = a(x-p)^2 + q$, which has a maximum point at the origin. Find the values of a, p and q.



Ans: $a =$ _____

$p =$ _____

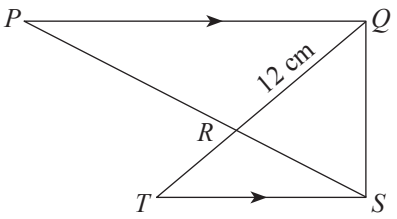
$q =$ _____ [4]

15 In the diagram, PRS and QRT are straight lines and PQ is parallel to TS. Given that the ratio $PR : PS = 3 : 5$, $QR = 12$ cm and area of $\triangle TSR = 92$ cm^2.

(a) Show that $\triangle QPR$ and $\triangle TSR$ are similar. State your reasons.

(b) Calculate the length TR.

(c) Find the area of $\triangle PQR$.



Ans: (a) on answer space cm [3]

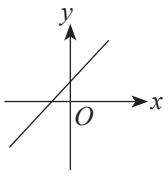
(b) $TR =$ _____ cm [1]

(c) _____ cm^2 [2]

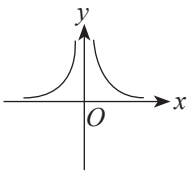
16 Which of the following graphs represent

(a) $y = 5^x$,

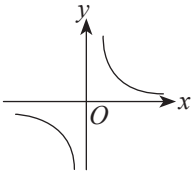
(b) $y = \frac{2}{x^2}$?



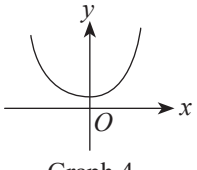
Graph 1



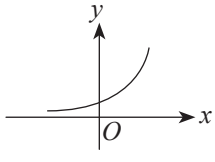
Graph 2



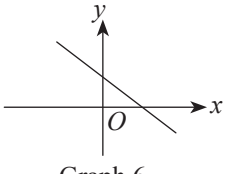
Graph 3



Graph 4



Graph 5



Graph 6

Ans: (a) Graph _____ [1]

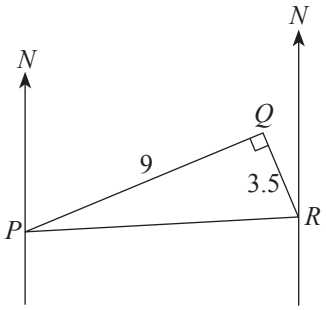
(b) Graph _____ [1]

17 The diagram shows three towns, P, Q and R. The bearing of P from Q is 245º. It is given that $\angle PQR = 90º$, $QR = 3.5$ km and $PQ = 9$ km. Find

(a) $\angle QPR$,

(b) the bearing of Q from P,

(c) the bearing of R from Q.



Ans: (a) $\angle QPR =$ _____ º [2]

(b) _____ º [2]

(c) _____ º [2]

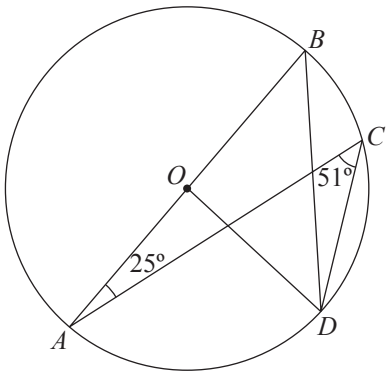
18 A credit card company charges a 5% monthly interest compounded on any outstanding debts. Alex owes the company \$3 000 at the end of January. He pays \$500 at the end of February. Find out how much he owes at the end of March.

Ans: \$ _____ [4]

19 In the diagram, O is the centre of the circle and AOB is the diameter. Given that $\angle BAC = 25°$ and $\angle ACD = 51°$, calculate

(a) $\angle ABD$,

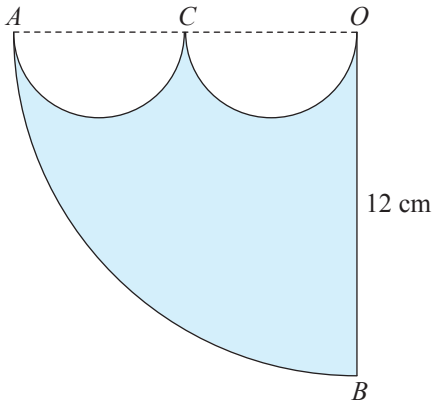
(b) $\angle DCB$.



Ans: (a) $\angle ABD =$ _____ ° [2]

(b) $\angle DCB =$ _____ ° [2]

20 The diagram shows a quadrant of a circle with centre O and radius 12 cm. Two identical semicircles are drawn along OA.
Find
(a) the perimeter and
(b) the area of the shaded region.
Give both answers in the form $a + b\pi$, where a and b are integers.



Ans: (a) _____ cm [2]

(b) _____ cm^2 [2]

Solutions to:

End of Year Examination Paper 1

1. $\sqrt[3]{a^9b^3} \times (4a^8)^0 = (a^9b^3)^{\frac{1}{3}} \times 1$ $\quad a^0 = 1$

$= a^3b$

2. $y = \dfrac{8x-3}{5}$

$y = \dfrac{8}{5}x - \dfrac{3}{5}$

$y = 1\dfrac{3}{5}x - \dfrac{3}{5}$ $\quad y = mx + c$

$\therefore$ Gradient $= 1\dfrac{3}{5}$, y-intercept $= -\dfrac{3}{5}$

3. (a) $\dfrac{0.00149}{100\,000} = 1.49 \times 10^{-8}$

$= 1.5 \times 10^{-8}$ (2 s.f.)

(b) (i) 2×10^{-9} m $= 0.002 \times 10^{-6}$ m

$= 0.002\ \mu$m

$= 2 \times 10^{-3}\ \mu$m

(ii) Number of DNA strands $= \dfrac{5 \times 10^{-2}\text{ m}}{2 \times 10^{-9}\text{ m}}$

$= 2.5 \times 10^7$

4. (a) $x(x+1) = 2$

$x^2 + x - 2 = 0$

$(x-1)(x+2) = 0$

$x - 1 = 0$ or $x + 2 = 0$

$x = 1$ or $x = -2$

(b) $\dfrac{3}{a-1} + \dfrac{4a}{1-3a} = 0$

$\dfrac{3(1-3a) + 4a(a-1)}{(a-1)(1-3a)} = 0$

$3(1-3a) + 4a(a-1) = 0$

$3 - 9a + 4a^2 - 4a = 0$

$4a^2 - 13a + 3 = 0$

$(a-3)(4a-1) = 0$

$a - 3 = 0$ or $4a - 1 = 0$

$a = 3$ or $a = \dfrac{1}{4}$

5. (a) $\dfrac{1}{4^x} = 2(16^{-x})$

$\dfrac{1}{2^{2x}} = 2(2^{-4x})$

$2^{-2x} = 2^{1-4x}$

Comparing indices,

$-2x = 1 - 4x$

$2x = 1$

$x = \dfrac{1}{2}$

(b) $\sqrt{3^y} \times 9^{y-1} \times \dfrac{1}{27^y} = 81$

$(3^y)^{\frac{1}{2}} \times 3^{2y-2} \times \dfrac{1}{3^{3y}} = 81$

$3^{\frac{y}{2}} \times 3^{2y-2} \times 3^{-3y} = 3^4$

$3^{\frac{y}{2} + (2y-2) + (-3y)} = 3^4$

$3^{-\frac{y}{2} - 2} = 3^4$

Comparing indices,

$-\dfrac{y}{2} - 2 = 4$

$-y - 4 = 8$

$y = -12$

6. (a) $3 - (5 - 2x) \geqslant 25$

$3 - 5 + 2x \geqslant 25$

$2x \geqslant 27$

$x \geqslant 13\dfrac{1}{2}$

(b) 17

7. (a) $\angle OQR = \angle OSR = 90°$ (tan. $\perp$ rad.)

Obtuse $\angle QOS$

$= 360° - 90° - 90° - x°$ ($\angle$ sum of quad. $OQRS$)

$= 180° - x°$

(b) $\angle QUS = \dfrac{180° - x°}{2}$ ($\angle$ at centre $= 2\angle$ at circumference)

or

$= 90° - \dfrac{x°}{2}$

(c) Let $\angle OQU = n°$

$\therefore \angle OSU = 2n°$

Reflex $\angle QOS = 360° - (180° - x°)$ ($\angle$s at a pt.)

$= 180° + x$

In quad. $OQUS$,

$n + \left(90 - \dfrac{x}{2}\right) + 2n + (180 + x) = 360$

$270 + 3n + \dfrac{x}{2} = 360$

$3n = 90 - \dfrac{x}{2}$

$n = 30 - \dfrac{x}{6}$

$\therefore \angle OQU = 30° - \dfrac{x°}{6}$

8. (a) Gradient $= \dfrac{7 - (-1)}{3 - 1}$

$= 4$

(b) Since $(k, 2)$ lies on line l, it forms a gradient of 4 with point $A(1, -1)$.

$\dfrac{2 - (-1)}{k - 1} = 4$

$3 = 4k - 4$

$4k = 7$

$k = 1\dfrac{3}{4}$

9. (a) Using the Cosine Rule on $\triangle ACD$,

$AC = \sqrt{70^2 + 100^2 - 2(70)(100)\cos 27°}$

≈ 49.2535 m

$= 49.3$ m (3 s.f.)

(b) Using the Sine Rule on $\triangle ABD$,

$\dfrac{100}{\sin \angle ABC} = \dfrac{60}{\sin 27°}$

$\sin \angle ABC \approx 0.75665$

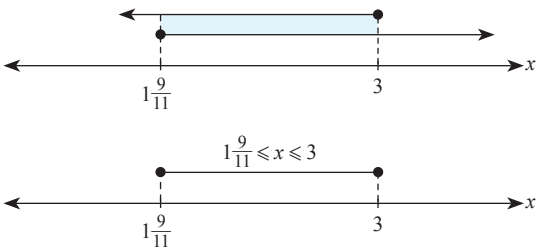
$\angle ABC \approx 49.170$

$= 49.2°$ (1 d.p.)

10. $-\frac{1}{4}x - 3 \geqslant 2 - 3x$ and $2 - 3x \geqslant x - 10$

$\frac{11}{4}x \geqslant 5$ and $4x \leqslant 12$

$x \geqslant 1\frac{9}{11}$ and $x \leqslant 3$



11. (a) $\triangle ABC = \triangle DEF$

(b) $EF = BC$ (corr. sides of congruent triangles)

$= 4$ cm

$\therefore x = EF - EC$

$= 4 \text{ cm} - 2.5 \text{ cm}$

$= 1.5$ cm

$y = AC$ (corr. sides of congruent triangles)

$= 3 \text{ cm} + 1.5 \text{ cm}$

$= 4.5$ cm

12. (a) By Pythagoras' Theorem on $\triangle CDF$,

$CF^2 = CD^2 + DF^2$

$= 4^2 + 12^2$

$= 160$

$CF = \sqrt{160}$

$= 12.6$ cm (3 s.f.)

(b) In $\triangle BFC$,

$\tan \angle BFC = \frac{BC}{CF}$

$= \frac{3}{\sqrt{160}}$

$\angle BFC = \tan^{-1} \frac{3}{\sqrt{160}}$

$= 13.3°$ (1 d.p.)

(c) By Pythagoras' Theorem on $\triangle BCF$,

$BF^2 = BC^2 + CF^2$

$= 3^2 + 160$

$= 169$

$BF = \sqrt{169}$

$= 13$ cm

$\therefore BO = AO = \frac{1}{2}(13)$

$= 6.5$ cm

Using the Cosine Rule on $\triangle AOB$,

$\cos \angle AOB = \frac{6.5^2 + 6.5^2 - 4^2}{2(6.5)(6.5)}$

≈ 0.81065

$\angle AOB = \cos^{-1} 0.81065$

$= 35.8°$ (1 d.p.)

13. $I = \frac{PRT}{100}$

$= \frac{86\,000 \times 1.15 \times 12}{100}$

$= \$11\,868$

Total amount $= \$86\,000 + \$11\,868$ Principal + Interest

$= \$97\,868$

14. $y = a(x - p)^2 + q$

Since the graph has a turning point at (0, 0),

$p = 0, q = 0$

$\therefore y = ax^2$

Subst. $y = -16, x = 4$ into $y = ax^2$,

$-16 = a(4)^2$

$a = -1$

$\therefore a = -1, p = 0$ and $q = 0$.

15. (a) $\angle QPR = \angle TSR$ (alt. $\angle$s, $PQ // TS$)

$\angle PQR = \angle STR$ (alt. $\angle$s, $PQ // TS$)

$\angle QRP = \angle TRS$ (vert. opp. $\angle$s)

$\therefore \triangle QPR$ is similar to $\triangle TSR$. (by AAA)

(b) Since $\triangle QPR$ and $\triangle TSR$ are similar,

$\frac{TR}{QR} = \frac{SR}{PR}$ corr. sides are proportional

$\frac{TR}{12} = \frac{2}{3}$

$TR = 8$ cm

(c) Let A_1 cm^2 be the area of $\triangle PQR$.

$\frac{A_1}{92} = \left(\frac{3}{2}\right)^2$ $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

$\frac{A_1}{92} = \frac{9}{4}$

$A_1 = 207$ cm^2

$\therefore$ The area of $\triangle PQR$ is 207 cm^2.

16. (a) Graph 5

(b) Graph 2

17. (a) In $\triangle PQR$,

$\tan \angle QPR = \frac{QR}{PQ}$

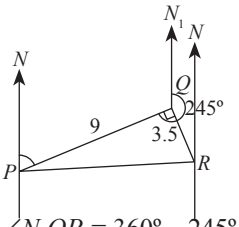
$= \frac{3.5}{9}$

$\angle QPR = \tan^{-1}\left(\frac{3.5}{9}\right)$

$\approx 21.2505°$

$= 21.3°$ (1 d.p.)

(b)



$\angle N_1QP = 360° - 245°$ ($\angle$s at a pt.)

$= 115°$

Bearing of Q from P
$= 180° - 115°$ (int. $\angle$s, $NP // N_1Q$)
$= 065°$ 3 digits for bearing

(c) Bearing of R from Q
$= \angle N_1QR$
$= 245° - \angle PQR$
$= 245° - 90°$
$= 155°$

18. Amount owed at the end of February
$= 3000\left(1 + \frac{5}{100}\right)^1$
$= \$3150$

Amount owed at the end of February after repaying \$500
$= \$3150 - \$500$
$= \$2650$
Amount owed at end of March
$= \$2650\left(1 + \frac{5}{100}\right)^1$
$= \$2782.50$

19. (a) $\angle ABD = 51°$ ($\angle$s in the same segment)
(b) $\angle ACB = 90°$ (rt. $\angle$ in semicircle)
$\angle DCB = 90° + 51°$
$= 141°$

20. (a) Diameter of semicircle $= 12 \div 2$
$= 6$ cm
Perimeter of shaded region
= Arc AB + Semicircle AC + Semicircle CO + radius OB
$= \frac{1}{4}\pi(24) + \frac{1}{2}\pi(6) + \frac{1}{2}\pi(6) + 12$
$= (12 + 12\pi)$ cm

(b) Area of quadrant $= \frac{1}{4}\pi(12)^2$
$= 36\pi$ cm^2
Area of semicircle $= \frac{1}{2}\pi(3)^2$
$= 4\frac{1}{2}\pi$ cm^2
Area of shaded region $= 36\pi - 4\frac{1}{2}\pi - 4\frac{1}{2}\pi$
$= 27\pi$ cm^2