Mid Year Examination Paper 1

INSTRUCTION TO CANDIDATES:

Marks Obtained
80

1. Answer **all** questions.
2. Write your answers and working in the spaces provided.
3. Omission of essential working will result in loss of marks.
4. Calculators may be used in this paper.
5. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer correct to three significant figures. Give answers in degrees correct to one decimal place.

Duration: 2 hours

1 (a) The number 90, written as a product of its prime factors, is $90 = 2 \times 3^a \times 5$. Write down the value of a.

(b) Find the value of b such that $\sqrt{b} = 135$, using prime factorisation. Give your answer in index notation.

Ans: (a) $a =$ _____ [1]

(b) $b =$ _____ [2]

2 Express the following in standard form.

(a) 0.000 040 18

(b) 615 000

Ans: (a) _____ [1]

(b) _____ [1]

3 A bank pays a simple interest of 6% per year. How much interest will be paid out on a $800 deposit in 3 years?

Ans: $ _____ [2]

4 Simplify the following expressions, expressing your answers in positive index notation.

(a) $\frac{8x^2}{7} \div \frac{x^3}{14}$

(b) $\left(\frac{ab}{3b^3}\right)^{-2}$

Ans: (a) _____ [2]

(b) _____ [2]

5 Find the values of x in the equation $a^{3x^2 - 11x + 6} = 1$, where $a > 0$.

Ans: $x =$ _____ or _____ [3]

6 (a) Express $x^2 - 6x - 5$ in the form $(x - b)^2 + c$.

(b) Hence, solve $x^2 - 6x - 5 = 0$, giving your answers correct to 2 decimal places.

Ans: (a) _____ [2]

(b) $x =$ _____ or _____ [3]

7 Given that $\frac{4x-7y}{5x-y}=\frac{1}{3}$, find the ratio of $x : y$.

Ans: _____ : _____ [2]

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8 Given that x and y are integers such that $-2 \leqslant x \leqslant 3$ and $2 \leqslant y \leqslant 4$, find

(a) the largest possible value of $y-x$,

(b) the smallest possible value of $\frac{x}{y}$,

(c) the least possible value of x^3-y.

Ans: (a) _____ [1]

(b) _____ [1]

(c) _____ [1]

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9 Solve for p and q in the following equations.

(a) $81 \times 3^{\frac{1}{2}} \div \sqrt[3]{3} = 3^p$

(b) $125 \div 5^{-7} = 5^q$

Ans: (a) p = _____ [3]

(b) q = _____ [3]

10 Solve the simultaneous equations by the substitution method.

$$\left.\begin{aligned} x + 4y &= 9 \\ 8y &= 17 - 3x \end{aligned}\right\}$$

Ans: $x =$ _____

$y =$ _____ [3]

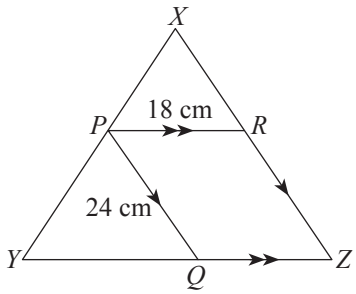
11 (a) Find the 5th term of a sequence given that the general term of the sequence is $T_n = 5 - 13n$.

(b) A different sequence U_n has the general term $U_n = 2T_n - 3n + 50$. Find the value of n when $U_n = -27$.

Ans: (a) $T_5 =$ _____ [1]

(b) $n =$ _____ [3]

12 In the diagram below, $PR//YZ$ and $PQ//XZ$. The side XY of $\triangle XYZ$ is divided such that PR is 75% that of YZ.



(a) Identify two different angles that $\angle XZY$ is equal to.

(b) Given that $PR = 18$ cm and $PQ = 24$ cm, find the length of XR.

Ans: (a) $\angle$ _____ and $\angle$ _____ [2]

(b) $XR =$ _____ cm [2]

13 (a) Solve $-3 \leqslant 2x - 4 < \frac{1}{2}(x + 2)$.

(b) State the largest integer which satisfies $-3 \leqslant 2x - 4 < \frac{1}{2}(x + 2)$.

Ans: (a) _____ [4]

(b) _____ [1]

14 Simplify the following algebraic fractions.

(a) $\left(\frac{14a^7}{4b^2}\right)\left(\frac{8ab}{7}\right)$

(b) $\frac{2}{x-3}+\frac{1}{(x-3)^2}$

(c) $\left(1 \div \frac{1}{p+1}\right) \div (p+1)^2$

Ans: (a) _____ [2]

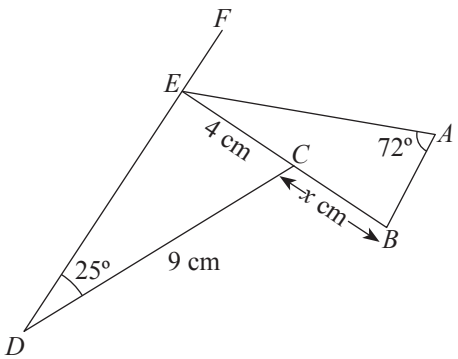
(b) _____ [2]

(c) _____ [2]

15 Given that triangle ABE is congruent to triangle ECD, $EC = 4$ cm, $CD = 9$ cm and $\angle CDE = 25°$, find

(a) the length of BC,

(b) $\angle AEF$.



Ans: (a) BC = _____ cm [1]

(b) $\angle AEF$ = _____ ° [1]

16 Steven drove from Singapore to Malacca at an average speed of $\left(\frac{x^2 + 4x - 45}{2x - 3}\right)$ km/h. The distance between the two cities is $\left(\frac{2x + 18}{2x - 3}\right)$ km. Find and simplify the expression for the time taken for the whole joumey in terms of x.

Ans: _____ h [4]

17 The masses of two similar metal discs are 500 g and 108 g respectively.

(a) Find the ratio of the diameter of the large disc to the diameter of the small disc.

(b) Given that the surface area of the large disc is 40 cm^2, find the surface area of the small disc.

Ans: (a) _____ : _____ [2]

(b) _____ cm^2 [2]

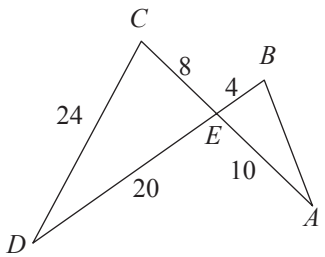
18 Solve the inequality $\frac{x+4}{4} - \frac{1}{2} < \frac{x}{2} + \frac{3x-5}{6}$

Ans: _____ [4]

19 The diagram below shows two triangles $\triangle ABE$ and $\triangle CDE$ such that AEC and BED are straight lines. $AE = 10$ cm, $BE = 4$ cm, $CE = 8$ cm, $CD = 24$ cm and $DE = 20$ cm.

(a) Show that triangles ABE and DCE are similar.

(b) Calculate the length AB.



Ans: (a) on answer space [3]

(b) AB = _____ cm [2]

20 James invested $30 000 in an investment which pays 3% interest per annum, compounded half-yearly. Find the total interest earned if the money is invested for 4 years.

Ans: $ _____ [3]

21 Two reservoirs A and B are similar in every aspect. The perimeter of reservoir A is 9 times that of reservoir B.

(a) Find $\frac{\text{capacity of reservoir } A}{\text{capacity of reservoir } B}$.

(b) Given that their total capacity is 292 megalitres, find the capacity of reservoir B, in standard form.

(1 megalitre = 1×10^6 litres)

Ans: (a) _____ [1]

(b) _____ litres [2]

22 Solve $\frac{1}{a+1}+\frac{1}{a+2}+\frac{1}{a+3}=0$

Ans: $a =$ _____ or _____ [3]

Solutions to:

Mid Year Examination Paper 1

1. (a)
$$\begin{array}{r|l} 2 & 90 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ \hline & 1 \end{array}$$
$90 = 2 \times 3^2 \times 5$
$\therefore a = 2$

(b)
$$\begin{array}{r|l} 3 & 135 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ \hline & 1 \end{array}$$
$135 = 3^3 \times 5$
$\sqrt{b} = 3^3 \times 5$
$b = 3^6 \times 5^2$ square both sides

2. (a) $0.000\,040\,18 = 4.018 \times 10^{-5}$
(b) $615\,000 = 6.15 \times 10^5$

3. Interest paid $= \frac{PRT}{100}$ $I = \frac{PRT}{100}$
$= \frac{800 \times 6 \times 3}{100}$
$= \$144$

4. (a) $\frac{8x^2}{7} \div \frac{x^3}{14} = \frac{8x^2}{7} \times \frac{14}{x^3}$
$= \frac{16}{x}$

(b) $\left(\frac{ab}{3b^3}\right)^{-2} = \left(\frac{3b^3}{ab}\right)^2$ $\left(\frac{p}{q}\right)^{-1} = \frac{q}{p}$
$= \frac{9b^6}{a^2b^2}$
$= \frac{9b^{6-2}}{a^2}$
$= \frac{9b^4}{a^2}$

5. $a^{3x^2-11x+6} = 1$
$a^{3x^2-11x+6} = a^0$ $a^0 = 1$
$\therefore 3x^2 - 11x + 6 = 0$ comparing indices
$(x-3)(3x-2) = 0$
$x - 3 = 0$ or $3x - 2 = 0$
$x = 3$ or $x = \frac{2}{3}$

6. (a) $x^2 - 6x - 5 = (x-3)^2 - 3^2 - 5$
$= (x-3)^2 - 14$
(b) $x^2 - 6x - 5 = 0$
$(x-3)^2 - 14 = 0$
$(x-3)^2 = 14$
$x - 3 = \pm\sqrt{14}$
$x = 3 + \sqrt{14}$ or $x = 3 - \sqrt{14}$
$= 6.74$ or $= -0.74$ (2 d.p.)

7. $\frac{4x-7y}{5x-y} = \frac{1}{3}$
$12x - 21y = 5x - y$
$7x = 20y$
$\frac{x}{y} = \frac{20}{7}$
$\therefore x : y = 20 : 7$

8. (a) Largest $(y - x) = 4 - (-2)$
$= 6$
(b) Smallest $\left(\frac{x}{y}\right) = \frac{-2}{2}$
$= -1$
(c) Least $(x^3 - y) = (-2)^3 - 4$
$= -12$

9. (a) $81 \times 3^{\frac{1}{2}} \div \sqrt[3]{3} = 3^p$
$3^4 \times 3^{\frac{1}{2}} \div 3^{\frac{1}{3}} = 3^p$
$3^{4+\frac{1}{2}-\frac{1}{3}} = 3^p$
$3^{\frac{25}{6}} = 3^p$
$\therefore p = 4\frac{1}{6}$ comparing indices

(b) $125 \div 5^{-7} = 5^q$
$5^3 \div 5^{-7} = 5^q$
$5^{3-(-7)} = 5^q$
$5^{10} = 5^q$
$q = 10$ comparing indices

10. $x + 4y = 9$ —— ①
$8y = 17 - 3x$ —— ②
From ①, $x = 9 - 4y$ —— ③
Subst. $x = 9 - 4y$ into ②: substitution method
$8y = 17 - 3(9 - 4y)$
$8y = 17 - 27 + 12y$
$-4y = -10$
$y = 2\frac{1}{2}$
Subst. $y = 2\frac{1}{2}$ into ③:
$x = 9 - 4\left(2\frac{1}{2}\right)$
$= -1$
$\therefore x = -1$ and $y = 2\frac{1}{2}$

11. (a) $T_n = 5 - 13n$

$T_5 = 5 - 13(5)$ subst. $n = 5$

$= -60$

(b) $U_n = 2T_n - 3n + 50$ $T_n = 5 - 13n$

$-27 = 2(5 - 13n) - 3n + 50$

$-27 = 10 - 26n - 3n + 50$

$-27 = 60 - 29n$

$29n = 87$

$n = 3$

12. (a) $\angle XZY = \angle XRP$ (corr. $\angle$s, $PR // QZ$)

Also,

$\angle XZY = \angle PQY$ (corr. $\angle$s, $RZ // PQ$)

(b) Since $PRZQ$ is a parallelogram, $RZ = 24$ cm.

$\therefore XZ = XR + 24$ cm

Since $\triangle XPR$ is similar to $\triangle XYZ$, prove it!

$\frac{XR}{XZ} = \frac{PR}{YZ}$ corr. sides are proportional

$\frac{XR}{XR + 24} = \frac{75\%}{100\%}$

$\frac{XR}{XR + 24} = \frac{3}{4}$

$4XR = 3XR + 72$

$XR = 72$ cm

13. (a) $-3 \leqslant 2x - 4 < \frac{1}{2}(x + 2)$

$-3 \leqslant 2x - 4$ and $2x - 4 < \frac{1}{2}(x + 2)$

$2x \geqslant 1$ $2x - 4 < \frac{1}{2}x + 1$

$x \geqslant \frac{1}{2}$ $x < 3\frac{1}{3}$

$\therefore \frac{1}{2} \leqslant x < 3\frac{1}{3}$

(b) 3

14. (a) $\left(\frac{14a^7}{4b^2}\right)\left(\frac{8ab}{7}\right) = \frac{4a^{7+1}}{b}$

$= \frac{4a^8}{b}$

(b) $\frac{2}{x-3} + \frac{1}{(x-3)^2} = \frac{2(x-3)+1}{(x-3)^2}$

$= \frac{2x-5}{(x-3)^2}$

(c) $\left(1 \div \frac{1}{p+1}\right) \div (p+1)^2 = p + 1 \times \frac{1}{(p+1)^2}$

$= \frac{1}{p+1}$

15. (a) $BC = BE - EC$

$= CD - 4$ cm $\because \triangle ABE \equiv \triangle ECD \Rightarrow BE = CD$

$= 9$ cm $- 4$ cm

$= 5$ cm

(b) Since $\triangle ABE \equiv \triangle ECD$,

$\angle AEB = \angle EDC$

$= 25°$

Also,

$\angle CED = \angle BAE$

$= 72°$

$\therefore \angle AEF = 180° - \angle AEB - \angle CED$ ($\angle$s on a st. line)

$= 180° - 25° - 72°$

$= 83°$

16. Time taken $= \frac{2x+18}{2x-3} \div \frac{x^2+4x-45}{2x-3}$ Time $= \frac{\text{Dist.}}{\text{Speed}}$

$= \frac{2x+18}{2x-3} \times \frac{2x-3}{x^2+4x-45}$

$= \frac{2x+18}{\cancel{2x-3}} \times \frac{\cancel{2x-3}}{(x-5)(x+9)}$

$= \frac{2x+18}{(x-5)(x+9)}$

$= \frac{2\cancel{(x+9)}}{(x-5)\cancel{(x+9)}}$

$= \frac{2}{x-5}$ h

17. (a) Since mass is proportional to volume, $\frac{V_1}{V_2} = \frac{500}{108}$.

$\frac{D_1}{D_2} = \sqrt[3]{\frac{V_1}{V_2}}$

$= \sqrt[3]{\frac{500}{108}}$

$= \frac{5}{3}$

Hence, the ratio is 5 : 3.

(b) $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

$\frac{40}{A_2} = \left(\frac{5}{3}\right)^2$

$\frac{40}{A_2} = \frac{25}{9}$

$A_2 = 14.4$ cm^2

The surface area of the smaller disc is 14.4 cm^2.

18. $\frac{x+4}{4} - \frac{1}{2} < \frac{x}{2} + \frac{3x-5}{6}$

$\frac{x+4-2}{4} < \frac{3x+(3x-5)}{6}$

$\frac{x+2}{4} < \frac{6x-5}{6}$

$\frac{3(x+2)}{12} < \frac{2(6x-5)}{12}$ common denominator = 12

$3(x+2) < 2(6x-5)$ multiply 12 on both sides

$3x + 6 < 12x - 10$

$-9x < -16$

$x > \frac{16}{9}$ reverse inequality sign

$x > 1\frac{7}{9}$

19. (a) $\frac{AE}{DE} = \frac{10}{20} = \frac{1}{2}$

$\frac{BE}{CE} = \frac{4}{8} = \frac{1}{2}$

$\therefore \frac{AE}{DE} = \frac{BE}{CE} = \frac{1}{2}$

Also

$\angle AEB = \angle DEC$ (vert. opp. $\angle$s)

$\therefore \triangle ABE$ is similar to $\triangle DCE$, by SAS. (shown)

(b) Since $\triangle ABE$ is similar to $\triangle DCE$,

$$\frac{AB}{DC} = \frac{AE}{DE}$$ corr. sides are proportional

$$\frac{AB}{24} = \frac{1}{2}$$

$$AB = 12 \text{ cm}$$

20. Compounding periods, $n = 4 \times 2$

$= 8$

Interest rate for each compounding period $= 3\% \div 2$

$= 1.5\%$

Total amount received $= 30\,000\left(1 + \frac{1.5}{100}\right)^8$

$= \$33\,794.78$ (2 d.p.)

Interest earned $= \$33\,794.78 - \$30\,000$

$= \$3794.78$

21. (a) $\frac{\text{Capacity of reservoir } A}{\text{Capacity of reservoir } B} = \left(\frac{l_1}{l_2}\right)^3$

$= \left(\frac{9}{1}\right)^3$

$= \frac{729}{1}$ or 729

(b) Capacity of reservoir $B = \frac{292 \text{ megalitres}}{729 + 1}$

$= 0.4$ megalitres

$= 0.4 \times 10^6$ litres

$= 4 \times 10^5$ litres

22. $$\frac{1}{a+1} + \frac{1}{a+2} + \frac{1}{a+3} = 0$$

$$\frac{(a+2)(a+3) + (a+1)(a+3) + (a+1)(a+2)}{(a+1)(a+2)(a+3)} = 0$$

$$(a+2)(a+3) + (a+1)(a+3) + (a+1)(a+2) = 0$$

$$a^2 + 5a + 6 + a^2 + 4a + 3 + a^2 + 3a + 2 = 0$$

$$3a^2 + 12a + 11 = 0$$

Using the quadratic formula,

$$a = \frac{-12 \pm \sqrt{12^2 - 4(3)(11)}}{2(3)}$$

$= -2.58$ or -1.42 (3 s.f.)