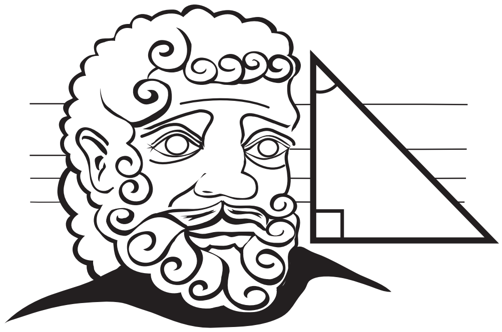
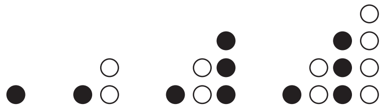
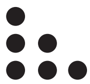
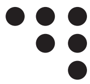
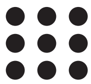
Number Patterns

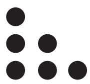
Ancient Greek mathematicians were interested in numbers made from different arrangements of counting objects. Numbers arranged in a triangular pattern suggested the sum of the counting numbers.





1 Putting 2 of the same triangular numbers gives the corresponding square number + that number; for example:

 +  is  +  or $3^2 + 3$

 is half of $[3^2 + 3]$

The 3rd triangular number, T_3, is half of $[3^2 + 3]$.

(a) Is this true for the 4th triangular number, T_4?

Answer: ____________________

(b) What happens when you add a triangular number and the next triangular number?

Answer: ____________________

(c) Show the reason for (b) using a diagram like the one in the previous page.

(d) Can you write this pattern using T_1, T_2, T_3, … for triangular numbers and S_2, S_2, S_3, … for square numbers?

Answer: ____________________

2 This arrangement of numbers is often called Pascal's triangle: each new entry is formed from the sum of the two numbers above it.

(a) Continue the pattern for two more rows.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

(b) Where are the triangular numbers?

Answer: ____________________

(c) Where do you find the sum of the triangular numbers?

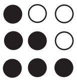
Answer: ____________________

Answers to:

Number Patterns

1 (a) **Yes**

(b) **It equates to a square number.**

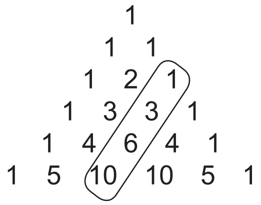
(c) 

(d) $T_2 + T_3 = S_3$

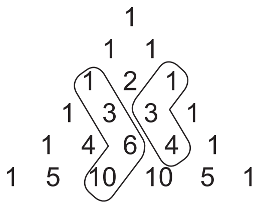
$T_3 + T_4 = S_4$, etc

2 (a)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

(b) 

The triangular numbers are found in the third diagonal.

(c) 

The sum of the triangular numbers are found diagonally below the last number.