Mid-year Examination Specimen Paper 1

1. (a) 90

(b) 89.5

2. (a) 0.12

(b) $12\% = \frac{12}{100} = \frac{3}{25}$

3. $3x - 4x(2x - 1) = 3x - 8x^2 + 4x$

$= 7x - 8x^2$

4.
$$n = 2\pi\sqrt{\frac{m}{g}}$$
$$\sqrt{\frac{m}{g}} = \frac{n}{2\pi}$$
$$\frac{m}{g} = \frac{n^2}{4\pi^2}$$
$$m = \frac{gn^2}{4\pi^2}$$

5. (a) $\frac{4a^2b^5}{6b^3c^3} = \frac{2a^2b^2}{3c^3}$

(b) $\frac{4}{2d+1} + \frac{1}{d-1} = \frac{4(d-1) + (2d+1)}{(2d+1)(d-1)}$

$= \frac{6d-3}{(2d+1)(d-1)}$

$= \frac{3(2d-1)}{(2d+1)(d-1)}$

6. (a) $3y - 12xy = 3y(1 - 4x)$

(b) $4z^2 - 25 = (2z)^2 - 5^2$

$= (2z + 5)(2z - 5)$

7.
$$4(m-1)^2 = 9m^2$$
$$[2(m-1)]^2 = (3m)^2$$
$$[2(m-1)^2] - (3m)^2 = 0$$

$2(m-1) + 3m = 0$ or $2(m-1) - 3m = 0$

$2(m-1) = -3m$ or $2(m-1) = 3m$

$5m = 2$ $\quad -m = 2$

$m = \frac{2}{5}$ $\quad m = -2$

$\therefore\ m = -2$ or $\frac{2}{5}$

8. (a) $m \propto \frac{1}{\sqrt{n}}$

$m = \frac{k}{\sqrt{n}}$ where k is a non-zero constant

Substitute $m = 4$ and $n = 9$ into $m = \frac{k}{\sqrt{n}}$.

$4 = \frac{k}{\sqrt{9}}$

$k = 12$

$\therefore\ m = \frac{12}{\sqrt{n}}$

(b) When $m = -6$,

$-6 = \frac{12}{\sqrt{n}}$

$-6\sqrt{n} = 12$

$\sqrt{n} = -2$

$n = (-2)^2$

$= 4$

9. 3.330

$\pi = 3.142 \ldots$

$\sqrt{10} = 3.162 \ldots$

$3.\dot{3} = 3.33333 \ldots$

$\therefore\ 3.\dot{3}, 3.330, \sqrt{10}, \pi$

10. (a) Sum of the ext. $\angle$s $= 360°$

$(n-2) \times 36 + 80 + 64 = 360$

$(n-2) \times 36 = 216$

$n - 2 = \frac{216}{36}$

$n - 2 = 6$

$n = 8$

(b) Largest int. $\angle = 180° - 36°$

$= 144°$

(Note: The largest int. $\angle$ corresponds to the smallest ext. $\angle$ in a polygon.)

11. $180° - (2x + 5°) = 175° - 2x$ (int. $\angle$s)

$175° - 2x + 260° + x = 360°$ ($\angle$s at a point)

$-x = -75°$

$x = 75°$

12. $4x - 3y = 8$ … (1)

$5y - 2x = -11$ … (2)

(2) × 2: $10y - 4x = -22$ … (3)

(1) + (3): $(4x - 3y) + (10y - 4x) = 8 + (-22)$

$7y = -14$

$y = -2$

Substitute $y = -2$ into (1):

$4x - 3(-2) = 8$

$x = 0.5$

$\therefore\ x = 0.5, y = -2$

13. (ai) $QR^2 = PR^2 + PQ^2$

$= 10^2 + 24^2$

$= 676$

$QR = 26$ cm ($QR > 0$)

(aii) $\frac{QS}{QR} = \frac{10}{13}$

$QS = \frac{10}{13} \times 26$

$= 20$ cm

$PS^2 = PQ^2 - QS^2$

$= 24^2 - 20^2$

$= 176$

$PS = 13.3$ cm (3 s.f.) ($PS > 0$)

(b) $\cos \angle PQR = \frac{24}{26}$

14. (a) $60 = 2^2 \times 3 \times 5$

$75 = 3 \times 5^2$

(b) $60n = 75k$ where k is a non-zero constant

$2^2 \times 3 \times 5 \times n = 3 \times 5^2 \times k$

$2^2 \times 3 \times 5 \times (5) = 3 \times 5^2 \times (2^2)$

$\therefore$ Least value of $n = 5$

15. (a) $\sqrt{5}, \sqrt{6}$

(b) $\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots, \ldots$

Ans: $\frac{6}{5} = 1\frac{1}{5}$, $\frac{7}{6} = 1\frac{1}{6}$

16. (a) $EC = \frac{1}{3}AB$

$= \frac{1}{3}(9)$

$= 3$ m

Area of $ABCE = 63$ m^2

$\frac{1}{2}(AB + EC) \times$ height $= 63$

$\frac{1}{2}(9 + 3) \times$ height $= 63$

$6 \times$ height $= 63$

Height $= 10.5$ m

Area of $\triangle AEC = \frac{1}{2} \times EC \times$ height

$= \frac{1}{2} \times 3 \times 10.5$

$= 15\frac{3}{4}$ m^2

(b) Area of the parallelogram $ABCD$

$= 2 \times$ (area of $ABCE$ – area of $\triangle AEC$)

$= 2\left(63 - 15\frac{3}{4}\right)$

$= 94.5$ m^2

Fraction

$= \dfrac{\text{area of the parallelogram} - \text{area of } \triangle AEC}{\text{area of the parallelogram}}$

$= \dfrac{94.5 - 15\frac{3}{4}}{94.5}$

$= \frac{5}{6}$

Mid-year Examination Specimen Paper 2

PART I

1. (a) Difference between Y and $X = (y - 2)$ units

Difference between Z and $X = 3$ units

3 units $\rightarrow$ 240 ml

$(y - 2)$ units $\rightarrow$ 560 ml

$\frac{y-2}{3} = \frac{560}{240}$

$\frac{y-2}{3} = \frac{7}{3}$

$y - 2 = 7$

$y = 9$

(b) $X : Y : Z = x + 2 : 9 : 5$

Total unit $= x + 16$

$(x + 16)$ units $\rightarrow$ 100%

$(x + 2)$ units $\rightarrow$ 30%

$\frac{x+2}{x+16} = \frac{30}{100}$

$\frac{x+2}{x+16} = \frac{3}{10}$

$10(x + 2) = 3(x + 16)$

$10x + 20 = 3x + 48$

$7x = 28$

$x = 4$

$\therefore$ 4 units of liquid X need to be added.

3 units $\rightarrow$ 240 ml

4 units $\rightarrow \frac{240}{3} \times 4$

$= 320$ ml

$\therefore$ 320 ml of liquid X need to be added.

2. (a) $BC = 3EC \Rightarrow \frac{BC}{EC} = \frac{3}{1}$

Since $\triangle ABC$ and $\triangle DEC$ are similar,

$\frac{DE}{AB} = \frac{EC}{BC}$

$\frac{DE}{6} = \frac{1}{3}$

$DE = 2$ cm

(b) $\frac{DC}{AC} = \frac{EC}{BC}$

$\frac{DC}{9 + DC} = \frac{1}{3}$

$3DC = 9 + DC$

$2DC = 9$

$DC = 4.5$ cm

(c) $\dfrac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEC} = \left(\frac{BC}{EC}\right)^2$

$\dfrac{\text{Area of } \triangle ABC}{8} = \left(\frac{3}{1}\right)^2$

Area of $\triangle ABC = 8 \times 9 = 72$ cm^2

Area of the quadrilateral $ABED = 72 - 8$

$= 64$ cm^2

3. $2x + 5y = 6$ … (1)

$4x - y = 1$

$y = 4x - 1$ … (2)

Substitute (2) into (1):

$2x + 5(4x - 1) - 6 = 0$

$22x - 11 = 0$

$x = \frac{1}{2}$

Substitute $x = \frac{1}{2}$ into (2):

$y = 4\left(\frac{1}{2}\right) - 1 = 1$

$\therefore x = \frac{1}{2}, y = 1$

4. (a) $p^2 + q^2 = 75$

$pq = 35$

$(p - q)^2 = p^2 - 2pq + q^2$

$= (p^2 + q^2) - 2pq$

$= 75 - 2(35)$

$= 5$

(b) $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$\frac{1}{3} + \frac{1}{v} = \frac{12}{7}$ $\left(\frac{1}{f} = \frac{12}{7}\right)$

$\frac{1}{v} = \frac{12}{7} - \frac{1}{3}$

$\frac{1}{v} = \frac{29}{21}$

$v = \frac{21}{29}$

(ci) 1.14×10^2
(cii) 9.79×10^{-5}

5. (a) 24 mm : 9.6 km
= 24 mm : 9600 m
= 24 mm : 9 600 000 mm
= 1 : 400 000

(b) $\frac{\text{Actual length of the tunnel}}{\text{Length of the tunnel on the map}} = \frac{9.6 \text{ km}}{2.4 \text{ cm}}$

$\frac{\text{Actual length of the tunnel}}{4.2 \text{ cm}} = \frac{9.6 \text{ km}}{2.4 \text{ cm}}$

Actual length of the tunnel $= \frac{9.6 \text{ km}}{2.4 \text{ cm}} \times 4.2$ cm
= 16.8 km

(c) $\frac{\text{Actual area}}{\text{Area on the map}} = \left(\frac{9.6 \text{ km}}{2.4 \text{ cm}}\right)^2$

$\frac{\text{Actual area}}{25 \text{ cm}^2} = \frac{9.6^2 \text{ km}^2}{2.4^2 \text{ cm}^2}$

Actual area $= \frac{9.6^2 \text{ km}^2}{2.4^2 \text{ cm}^2} \times 25 \text{ cm}^2$
$= 400 \text{ km}^2$

1 : 80 000
= 1 cm : 80 000 cm
= 1 cm : 0.8 km

$\frac{\text{Area on the new map}}{\text{Actual area}} = \left(\frac{1 \text{ cm}}{0.8 \text{ km}}\right)^2$

$\frac{\text{Area on the new map}}{400 \text{ km}^2} = \frac{1^2 \text{ cm}^2}{0.8^2 \text{ km}^2}$

Area on the new map $= \frac{1 \text{ cm}^2}{0.8^2 \text{ km}^2} \times 400 \text{ km}^2$
$= 625 \text{ cm}^2$

6. (a) $4(x-1)^2 = 25$
$2(x-1) = \pm 5$
$2x - 2 = -5$ or $2x - 2 = 5$
$x = -1.5$ $\quad$ $x = 3.5$
$\therefore x = -1.5, 3.5$

(b) $y(y-1) + 4y = 10$
$y^2 - y + 4y = 10$
$y^2 + 3y - 10 = 0$
$(y+5)(y-2) = 0$
$y + 5 = 0$ or $y - 2 = 0$
$y = -5$ $\quad$ $y = 2$
$\therefore y = -5, 2$

(c) $\frac{2}{z+3} - \frac{1}{z} = 0$

$\frac{2z - (z+3)}{z(z+3)} = 0$

$\frac{z-3}{z(z+3)} = 0$

$z - 3 = 0$
$z = 3$

7. (a) Let price of a pen = $\$x$
price of an eraser = $\$y$
$12x = 5y + 19.1$ … (1)
$60x + 25y = 120.5$ … (2)
(1) × 5: $60x = 25y + 95.5$
$60x - 25y = 95.5$ … (3)

(2) – (3):
$(60x + 25y) - (60x - 25y) = 120.5 - 95.5$
$50y = 25$
$y = 0.5$
Substitute $y = 0.5$ into (1):
$12x = 5(0.5) + 19.1$
$x = 1.8$
(i) A pen costs \$1.80.
(ii) An eraser costs \$0.50.

(b) Let the maximum number of pens she can buy be n.
$1.8n = 120.5$
$= 66\frac{17}{18}$
$\therefore$ She can buy at most 66 pens.

PART II

EITHER

(a) $\triangle ACD$ and $\triangle CBD$

(bi) Since $\triangle ACD$ is similar to $\triangle CBD$,
$\angle CAD = \angle BCD = 55°$

(bii) $AB^2 = AC^2 + CB^2$
$= 6.89^2 + 12^2$
$AB = 13.837$ $\quad (AB > 0)$
$= 13.8$ cm $\quad$ (3 s.f.)

(biii) Since $\triangle CBD$ is similar to $\triangle ABC$,

$\frac{\text{Area of } \triangle CBD}{\text{Area of } \triangle ABC} = \left(\frac{CB}{AB}\right)^2$

$\frac{\text{Area of } \triangle CBD}{\frac{1}{2} \times 6.89 \times 12} = \left(\frac{12}{13.837}\right)^2$

Area of $\triangle CBD = 31.092$
$= 31.1 \text{ cm}^2$ $\quad$ (3 s.f.)

(c) Let shortest distance = h cm.
$\frac{1}{2} \times BC \times h =$ area of $\triangle CBD$
$\frac{1}{2} \times 12 \times h = 31.092$
$h = 5.18$ cm $\quad$ (3 s.f.)

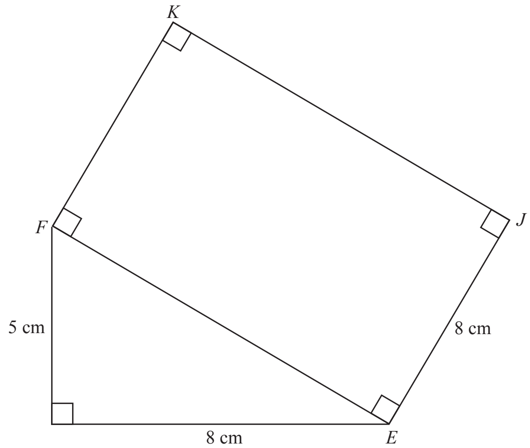
OR

(a) Area of $EFGH = \frac{1}{2} \times (FG + EH) \times GH$
$= \frac{1}{2}(2h + h)(8)$
$= 12h \text{ cm}^2$

(b) Volume of the prism = area of $EFGH \times HM$
$= 12h \times 8$
$= 96h \text{ cm}^3$
$96h = 12(3h^2 - 35)$
$8h = 3h^2 - 35$
$3h^2 - 8h - 35 = 0$ $\quad$ (shown)

(c) $3h^2 - 8h - 35 = 0$
$(h-5)(3h+7) = 0$
$h - 5 = 0$ or $3h + 7 = 0$
$h = 5$ $\quad$ $h = -\frac{7}{3}$ (rejected as $h > 0$)
$\therefore h = 5$

(d)



$EF^2 = 5^2 + 8^2$

$= 89$

$EF = \sqrt{89}$ cm $\quad (EF > 0)$

Area of $EFJK = EF \times EJ$

$= \sqrt{89} \times 8$

$= 75.5 \text{ cm}^2$

Final-year Examination Specimen Paper 1

1. (a) $\frac{4}{25} = 0.16$

(b) $\frac{4}{25} \times 100\% = 0.16 \times 100\% = 16\%$

2. (a) $\frac{6x^2yz^3}{8xz} \div \left(\frac{y^2}{4x^3z}\right)^2 = \frac{3xyz^2}{4} \div \frac{y^4}{16x^6z^2}$

$= \frac{3xyz^2}{4} \times \frac{16x^6z^2}{y^4}$

$= \frac{12x^7z^4}{y^3}$

(b) $\frac{4}{m^2 + 2mn - 3n^2} - \frac{1}{m-n}$

$= \frac{4}{(m+3n)(m-n)} - \frac{1}{m-n}$

$= \frac{4-(m+3n)}{(m+3n)(m-n)}$

$= \frac{4-m-3n}{(m+3n)(m-n)}$

3. (a) $9a^2 - 49b^2 = (3a)^2 - (7b)^2$

$= (3a - 7b)(3a + 7b)$

(b) $3p - q - 6pq + 2q^2 = (3p - q) - 2q(3p - q)$

$= (3p - q)(1 - 2q)$

4. $a^2 + 2ab = b$

$b - 2ab = a^2$

$b(1 - 2a) = a^2$

$b = \frac{a^2}{1-2a}$

5. $\frac{x+20}{x+2} = 2x$

$x + 20 = 2x(x + 2)$

$x + 20 = 2x^2 + 4x$

$2x^2 + 3x - 20 = 0$

$(2x - 5)(x + 4) = 0$

$2x - 5 = 0$ or $x + 4 = 0$

$x = 2.5 \qquad x = -4$

$\therefore x = -4, 2.5$

6. 2 cm : 15 km

(a) $\frac{\text{Actual distance}}{\text{Distance on the map}} = \frac{15 \text{ km}}{2 \text{ cm}}$

Actual distance $= \frac{15 \text{ km}}{2 \text{ cm}} \times 11$ cm

$= 82.5$ km

(b) $\frac{\text{Area on the map}}{\text{Actual area}} = \left(\frac{2 \text{ cm}}{15 \text{ km}}\right)^2$

Area on the map $= \frac{4 \text{ cm}^2}{225 \text{ km}^2} \times 450 \text{ km}^2$

$= 8 \text{ cm}^2$

7. (a) Gradient = 5

(b) $c = y\text{-intercept}$
$= -3$

(c) $y = 5x - 3$

At the x-axis, $y = 0$

$5x - 3 = 0$

$x = 0.6$

$\therefore P = (0.6, 0)$

8. $5x - 4y - 1.5 = 0$ … (1)

$x - 2y = 0$ … (2)

From (2): $x = 2y$ … (3)

Substitute (3) into (1):

$5(2y) - 4y - 1.5 = 0$

$6y = 1.5$

$y = 0.25$

Substitute $y = 0.25$ into (3):

$x = 2(0.25)$

$= 0.5$

$\therefore x = 0.5, y = 0.25$

9. (a) $d \propto \frac{1}{w}$

$d = \frac{k}{w}$ where k is a constant

Substitute $d = 4$ and $w = 250$ into $d = \frac{k}{w}$.

$4 = \frac{k}{250}$

$k = 1000$

$\therefore d = \frac{1000}{w}$

(b) 8000 toys → 250 workers × 4 days

16 000 toys → 400 workers × d days

$\frac{d \times 400}{4 \times 250} = \frac{16\,000}{8000}$

$d = \frac{16\,000}{8000} \times \frac{4 \times 250}{400}$

$= 5$

$\therefore$ 5 days are needed.

(c) 250 workers × 4 days → 8000 toys

500 workers × 10 days → n toys

$\frac{n}{8000} = \frac{500 \times 10}{250 \times 4}$

$n = \frac{500 \times 10}{250 \times 4} \times 8000$

$= 40\,000$

$\therefore$ 40 000 toys will be produced.

10. (a) Sum of the 10 integers = mean × 10

= 75 × 10

= 750

Mean of the 7 integers = 72 × 7

= 504

Sum of the other 3 integers = 750 – 504

= 246

Mean of the 3 integers $= \frac{246}{3}$

= 82

(b) Sum of the 9 integers

= 76 + 70 + 65 + 60 + 80 + 67 + 78 + 89 + 90

= 675

Last integer = 750 – 675

= 75

60, 65, 67, 70, <u>75</u>, <u>76</u>, 78, 80, 89, 90

Position of median $= \left(\frac{10 + 1}{2}\right)$th

$= 5^{\text{th}}$ and 6^{th}

$\therefore$ Median $= \frac{75 + 76}{2}$

= 75.5

11. Box A: Number of yellow marbles = 9

Number of blue marbles = 25 – 9

= 16

Box B: Number of black marbles = 15

Number of blue marbles = 10

(ai) P (blue) $= \frac{16}{25}$

(aii) P (red) $= \frac{0}{25}$

= 0

(b) Bag: Number of blue marbles = 16 + 10

= 26

Total number of marbles = 25 + 15 + 10

= 50

P (blue) $= \frac{26}{50}$

$= \frac{13}{25}$

12. (a) $LN^2 = MN^2 - LM^2$

$= 5^2 - 3^2$

$= 16$

$LN = \sqrt{16}$ $(LN > 0)$

= 4 cm

(b) $KL = \frac{3}{2} LN$

$= \frac{3}{2} \times 4$

= 6 cm

$KM^2 = ML^2 + KL^2$

$= 3^2 + 6^2$

$= 45$

$KM = \sqrt{45}$ $(KM > 0)$

= 6.71 cm (3 s.f.)

(c) Area of $\triangle KMN$

$= \frac{1}{2} \times NK \times ML$

$= \frac{1}{2} \times (6 - 4) \times 3$

$= 3 \text{ cm}^2$

Final-year Examination Specimen Paper 2

PART I

1. (a) $LN^2 = KN^2 + KL^2$
$= 7^2 + 24^2$
$= 625$
$LN = \sqrt{625}$ $(LN > 0)$
$= 25$ cm
$NP = \frac{2}{5} LN$
$= \frac{2}{5} \times 25$
$= 10$ cm

(b) $\frac{KL}{NP} = \frac{24}{10}$
$= \frac{12}{5}$

(c) Since $KLMN$ is similar to $NPQR$,
$\frac{NQ}{KM} = \frac{NP}{KL}$
$\frac{NQ}{25} = \frac{5}{12}$
$NQ = \frac{5}{12} \times 25$
$= 10\frac{5}{12}$
$= 10.4$ cm

(d) $\frac{\text{Area of } NPQR}{\text{Area of } KLMN} = \left(\frac{NP}{KL}\right)^2$
$\frac{\text{Area of } NPQR}{7 \times 24} = \left(\frac{5}{12}\right)^2$
Area of $NPQR = \frac{25}{144} \times 168$
$= 29\frac{1}{6}$
$= 29.2$ cm^2

(e) Area of $PQML$
= Area of $\triangle LMN$ – area of $\triangle NPQ$
$= \frac{1}{2} \times 7 \times 24 - \frac{1}{2} \times 29\frac{1}{6}$
$= 69.4$ cm^2

2. (a) $BD = 4 + 5 = 9$ m
Using Pythagoras' Theorem,
$AB^2 + BD^2 = AD^2$
$12^2 + 9^2 = AD^2$
$AD^2 = 225$
$AD = \sqrt{225}$
$= 15$ m

(b) (i) $\tan \angle ACB = \frac{9}{4}$

(ii) $\cos \angle ADB = \frac{12}{15}$
$= \frac{4}{5}$

(c) $\angle BAD$
Note: $\sin \angle BAD = \frac{12}{15}$
$\Rightarrow \angle BAD = \sin^{-1}\left(\frac{12}{15}\right)$

3. (a) $AD = \frac{12}{13}AC$
$13AD = 12AC$
$13(4y) = 12(3x - 2)$
$52y = 36x - 24$
$13y = 9x - 6$

(b) $AB = AC$
$2x + y = 3x - 2$
$y = x - 2$

(c) $13y = 9x - 6$ … (1)
$y = x - 2$ … (2)
Substitute (2) into (1):
$13(x - 2) = 9x - 6$
$13x - 26 = 9x - 6$
$4x = 20$
$x = 5$
Substitute $x = 5$ into (2):
$y = 5 - 2$
$= 3$
$\therefore x = 5, y = 3$

(d) $AD = 4(3)$
$= 12$ cm
$AC = 3(5) - 2$
$= 13$ cm
$DC^2 = AC^2 - AD^2$
$= 13^2 - 12^2$
$= 25$
$DC = \sqrt{25}$ $(DC > 0)$
$= 5$ cm

4. (a) Modal class = (12 – 13) minutes

(b) Position of median $= \left(\frac{50 + 1}{2}\right)$th
= 25th and 26th
Median class = (14 – 15) minutes

(c) Mean
$= \frac{3(10.5)+18(12.5)+12(14.5)+9(16.5)+6(18.5)+2(20.5)}{50}$
$= \frac{731}{50}$
$= 14.62$
$= 14.6$ minutes (3 s.f.)

(d) P (time = 16.5 min) $= \frac{9}{50}$

(e) P (time > 13 min) $= \frac{12 + 9 + 6 + 2}{50}$
$= \frac{29}{50}$

5. (ai) When $x = -0.5$, $a = 2(-0.5)^2 - 3(-0.5) - 2$
$= 0$

(aii) When $x = 1.5$, $b = 2(1.5)^2 - 3(1.5) - 2$
$= -2$

(b) **(See diagram 5. (b) on page S8)**

(ci) From the graph, when $x = 1.2$,

$y = -2.75$

(cii) $y = 2x^2 - 3x - 2$

$2x^2 - 3x - 2 = 0$

$y = 0$

From the graph, when $y = 0$,

$x = -0.5, 2.0$

PART II

EITHER

(a) Volume of the cylinder $= \pi \times \text{radius}^2 \times \text{height}$

$= \pi \times r^2 \times h$

$= \pi r^2 h$

Volume of the sphere $= \frac{4}{3}\pi \times \text{radius}^3$

$= \frac{4}{3}\pi(2r)^3$

$= \frac{32}{3}\pi r^3$

Volume of the cylinder $= \frac{1}{4} \times$ volume of the sphere

$\pi r^2 h = \frac{1}{4} \times \frac{32}{3}\pi r^3$

$h = \frac{8}{3}r$

(b) Surface area of the cylinder

$= 2\pi \times \text{radius}^2 + 2\pi \times \text{radius} \times \text{height}$

$= 2\pi r^2 + 2\pi rh$

Surface area of the sphere

$= 4\pi \times \text{radius}^2$

$= 4\pi(2r)^2$

$= 16\pi r^2$

S.A. of the sphere = S.A. of the cylinder + $(8r^2 + 6)\pi$

$16\pi r^2 = 2\pi r^2 + 2\pi rh + (8r^2 + 6)\pi$

$16r^2 = 10r^2 + 2rh + 6$

$2rh = 6r^2 - 6$

$h = \frac{3r^2 - 3}{r}$ (shown)

(ci) $h = \frac{8}{3}r$ … (1)

$h = \frac{3r^2 - 3}{r}$ … (2)

Substitute (1) into (2):

$\frac{8}{3}r = \frac{3r^2 - 3}{r}$

$8r^2 = 9r^2 - 9$

$r^2 = 9$

$r = 3$ or $r = -3$ (rejected as $r > 0$)

$\therefore r = 3$

(cii) When $r = 3$, $h = \frac{8}{3}(3) = 8$

(ciii) Volume of the cylinder $= \pi(3)^2(8)$

$= 72\pi$

$= 226 \text{ cm}^3$ (3 s.f.)

OR

(a) Volume $= \frac{\text{mass}}{\text{density}}$

$= \frac{3000 \text{ g}}{5 \text{ g/cm}^3}$

$= 600 \text{ cm}^3$

(b) Volume of the hollow cylinder $= \pi(r_1^2 - r_2^2)h$

$= \pi(14^2 - x^2)h$

$= \frac{22}{7}(14^2 - x^2)h$

Volume $= 600 \text{ cm}^3$

$\frac{22}{7}(14^2 - x^2)h = 600$

$14^2 - x^2 = \frac{600}{\frac{22}{7}h}$

$196 - x^2 = \frac{2100}{11h}$

$x^2 = 196 - \frac{2100}{11h}$

$x = \sqrt{196 - \frac{2100}{11h}}$

(c) Volume of the pyramid $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 72 \times \frac{1}{2}h$

$= 12h$

(di) Volume of the pyramid = volume of the cylinder

$12h = 600$

$h = 50 \text{ cm}$

(dii) $x = \sqrt{196 - \frac{2100}{11(50)}}$

$= 13.863 \text{ cm}$

$= 13.9 \text{ cm}$ (3 s.f.)

(e) $r_1 = 28 \div 2$

$= 14 \text{ cm}$

$r_2 = x$

$= 13.863 \text{ cm}$

$= 2\pi(r_1 + r_2)h + 2\pi(r_1^2 - r_2^2)$

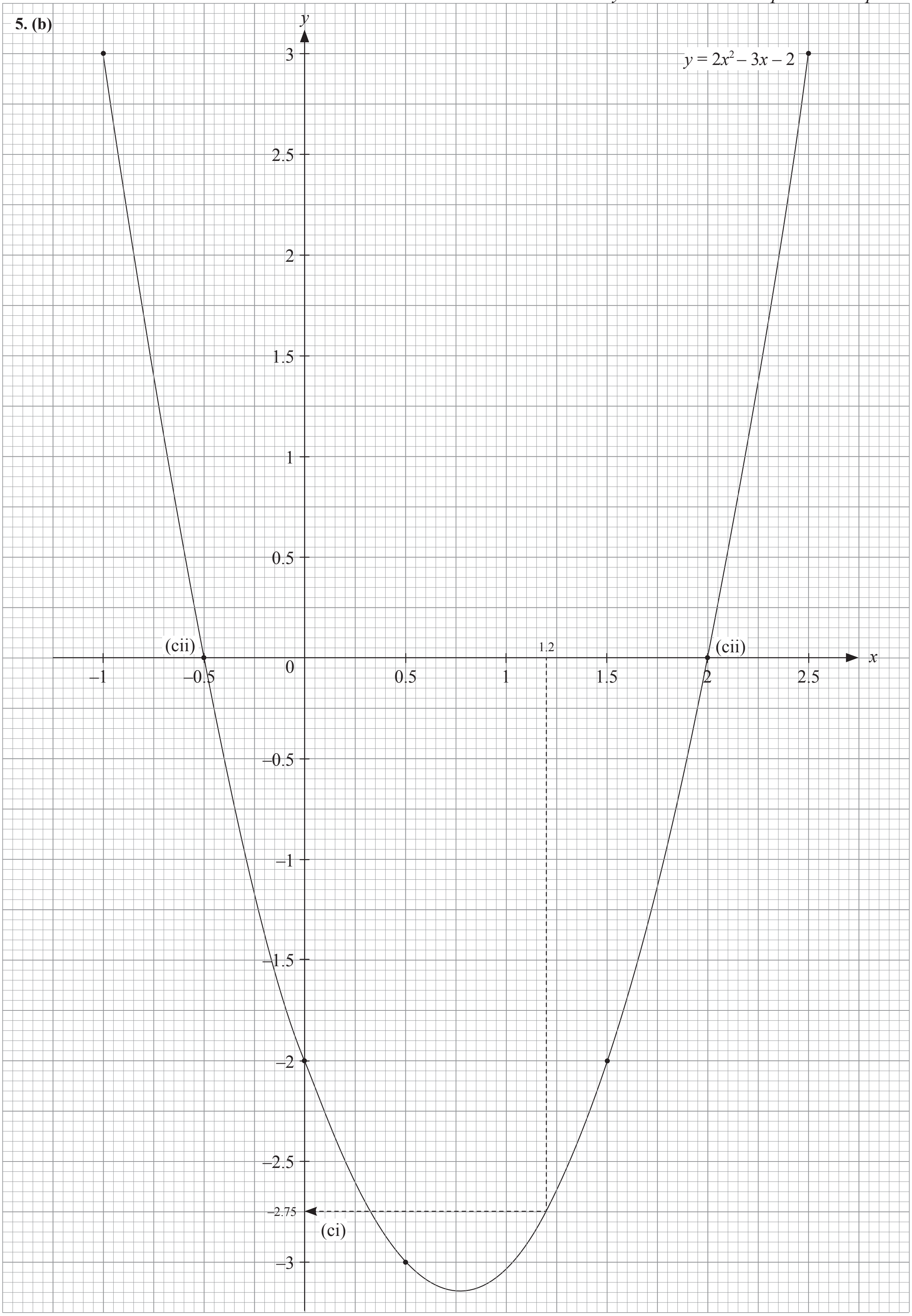
Total surface area of the hollow cylinder

$= 2\left(\frac{22}{7}\right)(14 + 13.863)(50) + 2\left(\frac{22}{7}\right)(14^2 - 13.863^2)$

$= 8780.9$

$= 8781 \text{ cm}^2$ (3 s.f.)

5. (b)


y
x
y = 2x² – 3x – 2
3
2.5
2
1.5
1
0.5
0
–0.5
–1
–1.5
–2
–2.5
–2.75
–3
–1
–0.5
0.5
1
1.2
1.5
2
2.5
(cii)
(cii)
(ci)